



IDENTIFYING DATA

Mathematics: Linear algebra

Subject	Mathematics: Linear algebra			
Code	O06G151V01106			
Study programme	Grado en Ingeniería Informática			
Descriptors	ECTS Credits	Choose	Year	Quadmester
	6	Basic education	1st	2nd
Teaching language	#EnglishFriendly Spanish			
Department				
Coordinator	Faro Rivas, Emilio			
Lecturers	Castro Vidal, Alberto de Faro Rivas, Emilio			
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General description	This subject belongs to the area of Mathematics and it is offered in the second semester of the first year. The subject has a character of basic education.			

The lectures are given in Spanish.

English Friendly subject: International students may request from the teachers: a) materials and bibliographic references in English, b) tutoring sessions in English, c) exams and assessments in English.

Training and Learning Results

Code				
A2	Students will be able to apply their knowledge and skills in their professional practice or vocation and they will show they have the required expertise through the construction and discussion of arguments and the resolution of problems within the relevant area of study.			
A3	Students will be able to gather and interpret relevant data (normally within their field of study) that will allow them to have a reflection-based considered opinion on important issues of social, scientific and ethical nature.			
B8	Knowledge of the essential subjects and technologies that will allow students to learn and develop new methods and technologies, as well as those that will endow them with versatility to adapt to new situations.			
B9	Ability to solve problems by taking the initiative, making decisions and acting independently and creatively. Ability to communicate the knowledge contents, skills and abilities of the Computer Science Engineer profession.			
C1	Ability to solve mathematical problems that might arise in engineering. Ability to apply knowledge of: linear algebra; integral and differential calculus; numerical methods; numerical algorithms; statistics and optimization.			
C3	Ability to understand and master the essential concepts of discrete mathematics, mathematical logic, algorithmic mathematics and computational complexity, and their application to the resolution of engineering problems.			
C12	Knowledge and application of basic algorithmic procedures of computer technologies to design solutions to problems, analyzing the appropriacy and complexity of the proposed algorithms.			
D4	Analysis, synthesis and evaluation capacity			
D5	Organizational and planning skills			
D6	Ability to abstract: ability to create and use models that reflect real situations			
D7	Ability to search, relate and structure information from various sources and to integrate ideas and knowledge.			
D11	Critical thinking			

Expected results from this subject

Expected results from this subject	Training and Learning Results			
RA1. To know how to use gaussian elimination to find an echelon form and the reduced echelon form of a matrix.	A2	B8	C1 C3 C12	D4 D6 D11

RA2. To understand and to know how to solve the questions of existence, uniqueness and universal existence for the systems of linear equations.	A2	B8	C1	D4 D6 D11
RA3. To understand the matrix product and its relationship with the composition of linear maps as well as to know its algebraic properties and its applications.	A2	B8	C1	D4 D6 D11
RA4. To understand what means for a matrix to have a right inverse, a left inverse or being invertible.	A2	B8	C1	D4 D6 D11
RA5. To know how to operate with block matrices and to know its properties and applications.	A3	B8 B9	C1 C3	D4 D6 D7 D11
RA6. To understand the concept of determinant of a square matrix, its properties and how to use those properties to calculate a determinant. To know how to calculate a determinant by the method of cofactors.	A2	B8	C1	D4 D6 D11
RA7. To understand the concept of vector space and that of linear map as well as the relationship between the concepts of kernel and image of a linear map and those of null space and column space of a matrix.	A2	B8	C1	D4 D6 D11
RA8. To understand the relationship between the questions of universal existence and uniqueness of solutions of a system of linear equations and the questions of subspace generated by and linear independence of the columns of a matrix, as well as the relationship between those and the properties of surjectivity and injectivity of a linear map.	A2	B8	C1	D4 D6 D11
RA9. To find a basis of the null space / column space of a matrix or of the kernel / image space of a linear map.	A2	B8	C1	D4 D6 D11
RA10. To find the cartesian equations of a subspace determined by means of generators and to find a basis and the cartesian equations of the sum or intersection of two subspaces of \mathbb{R}^n .	A2	B8	C1	D4 D6 D11
RA11. To find the coordinates of a vector with respect to a given basis and to find the change of coordinates matrix from a given basis to another one.	A2	B8	C1	D4 D6 D11
RA12. To know how to use coordinates to translate problems in abstract vector spaces to problems in \mathbb{R}^n .	A2	B8	C1	D4 D6 D11
RA13. To find the matrix of an endomorphism of a vector space relative to a given basis and to know how to determine the effect of a change of basis on the matrix of the endomorphism.	A2	B8	C1	D4 D6 D11
RA14. To understand the concept of diagonalization of a square matrix and its application to the calculation of powers of a square matrix and, in general, to the evaluation of a polynomial function on a square matrix.	A2	B8	C1	D4 D6 D11
RA15. To understand the concept of eigenvector and eigenvalue of a square matrix.	A2	B8	C1	D4 D6 D11
RA16. To know how to find the characteristic polynomial of a square matrix, its relationship with the eigenvalues and the spectrum of the matrix and the concept of algebraic multiplicity of the eigenvalues.	A2	B8	C1	D4 D6 D11
RA17. To know how to find a basis of the eigenspace of an eigenvalue of a square matrix and to know how to find a diagonalization of a matrix whose eigenvalues are known.	A2	B8	C1	D4 D6 D11
RA18. To understand the concepts of scalar product and orthogonality in \mathbb{R}^n and to understand the null space of a matrix as the orthogonal space to the row space of the matrix.	A2	B8	C1	D4 D6 D11
RA19. To calculate the orthogonal projection of a vector on the ray of a nonzero vector and to know how to use such projections to orthogonalize a basis of a subspace of \mathbb{R}^n by the Gram-Schmidt algorithm.	A2	B8	C1 C12	D4 D6 D11
RA20. To understand the problem of least squares associated with an inconsistent system of linear equations and to solve it by means of the corresponding normal equations.	A2	B8	C1	D4 D6 D11
RA21. To know the orthogonality properties of the eigenspaces of a symmetric matrix and to know how to use them to find an orthogonal diagonalization of a symmetric matrix.	A2	B8	C1	D4 D6 D11
RA22. To understand the concept of quadratic form and to know how to represent it by means of a symmetric matrix.	A2	B8	C1	D4 D6 D11
RA23. To understand the concept of change of variable for a quadratic form and to know how to find its effect on the corresponding symmetric matrix.	A2	B8	C1	D4 D6 D11

RA24. To know how to find a diagonalization of a quadratic form and to know how to use it to classify it and to determine its maximum and minimum values on unit vectors.

A2 B8 C1 D4
D5
D6
D11

Contents

Topic

BLOCK I

SYSTEMS OF LINEAR EQUATIONS:

Elementary row operations.
Echelon form and Reduced Echelon Form.
Vector equations.
Matrix equations and homogeneous systems.

LINEAR MAPS

Linear independence and linear maps.
The questions of existence and uniqueness in terms of linear maps.

MATRICES:

Matrix product. LU factorisation.
Invertible matrices.
Partitioned matrices.
Subspaces and basis.
Dimension and Rank.

BLOCK II

DETERMINANTS:

Definition of determinants and cofactors.
Calculation by elementary operations.
Applications of determinants.

VECTOR SPACES:

Definition and examples of vector space.
Coordinates. Vector subspaces.
Linear maps and their associated subspaces.
The matrix of a linear map and change of basis.
Similar matrices.

DIAGONALIZATION:

Eigenvectors and eigenvalues.
Eigenspace of an eigenvalue.
Characteristic polynomial.
Diagonalizable matrices and applications.

BLOCK III

ORTHOGONALITY AND LEAST SQUARES:

Inner product spaces and orthogonality.
Orthogonal projection on a subspace.
Gram-Schmidt algorithm and QR decomposition.
Least Squares problems.

SYMMETRIC MATRICES AND QUADRATIC FORMS:

Orthogonal diagonalization of symmetric matrices.
Quadratic forms.

LABORATORY PRACTICES

- Systems of linear equations.
- Matrix calculations.
- Geometric applications in the plane and in space.
- Matrix diagonalization.
- Inner product spaces.
- Classification of quadratic forms.

Planning

	Class hours	Hours outside the classroom	Total hours
Lecturing	23	36	59
Problem solving	16.5	24.75	41.25
Problem and/or exercise solving	10	11.75	21.75
Problem and/or exercise solving	1	9	10
Self-assessment	6	0	6

Problem and/or exercise solving	3	9	12
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*The information in the planning table is for guidance only and does not take into account the heterogeneity of the students.

Methodologies	
	Description
Lecturing	Lecture on the subject contents by the teacher, illustrated with numerous examples and applications.
Problem solving	Study, analysis and resolution of one or several problems or exercises related with the previously given topics. The said problems or exercises will illustrate or will complete the explanation of each lesson.
	Simultaneously, there will be proposed exercises and problems that the students will have to solve. The answers will be explained and the qualification obtained by each student will be part of the continuous evaluation.

Personalized assistance	
Methodologies	Description
Problem solving	In the tutorials those students that need a more personalized explanation of any aspect of the subject will be attended.
Lecturing	In the tutorials those students that need a more personalized explanation of any aspect of the subject will be attended.

Assessment					
	Description	Qualification	Training and Learning Results		
Problem and/or exercise solving	A weekly test will be given in the recitation classes. In those tests the learning outcomes RA1 to RA23 will be evaluated.	10	B8	C1	
Problem and/or exercise solving	One midterm exam will be given. The learning outcomes to be evaluated are:	35	B8	C1	
	□ RA1 to RA13.				
Self-assessment	(*)Realización de 6 pruebas de autoevaluación.	15	A2 A3	B9	C1 C3 C12 D4 D5 D6 D7 D11
Problem and/or exercise solving	Final examination at the end of the semester.	40	A2 A3	B8 B9	C1 C3 C12 D4 D5 D6 D7 D11
	Learning outcomes to be evaluated: RA1 to RA24.				

Other comments on the Evaluation

EVALUATION TESTS

There will be four types of tests:

1. A midterm exam (PMC),
2. Six self-evaluation tests (PAQ),
3. Ten True/Fase tests (CP) y
4. A Final exam at the end of the semester (June) and its second opportunity (July) in the dates fixed by the School.

MODES OF GRADING

Two modes:

- 1.- CONTINUOUS GRADING: The midterm counts 35%; the six self-evaluation tests will count together 15%; the ten True/False tests will count together 10% the final will count 40%.
- 2.- GLOBAL GRADING: The final exam will count 100%.

Choice of mode of evaluation (continuos/global)

Each student can choose the mode of evaluation better for him or her any time during the semester even after the grades of the final exam have come out.

DEFAULT MODE OF EVALUATION

TWO GRADES WILL BE CALCULATED FOR EACH STUDENT AND THE HIGHEST OF THE TWO WILL BE ASSIGNED BY DEFAULT.

FORMULA FOR THE FINAL GRADE

The default final grade will be calculated by the following formula:

$$NF = \max (0,6*PEC + 0,4*EF , EF + (3/50)*PEC*(10 - EF))$$

where PEC is the weighted average of the three grades of continuous evaluation (in the range 0-10): PMC, MAQ, and CP with weights as indicated above.

Evaluated competencies: CB2, CB3, CG8, CG9, CE1, CE3, CE12, CT4, CT5, CT6, CT7, CT11.

Evaluated learning outcomes: RA1 to RA24.

CRITERIA OF EVALUATION FOR END OF CAREER EXAM

Methodology/Single Test: Evaluation of theory and problems.

Description: Written exam that will include evaluation of theoretical concepts and resolution of exercises.

% Qualification: 100%.

PROCESS OF ASSIGNING THE FINAL GRADE

Independently of the announcement, the final grade will be the symmetrical round to 1 decimal places of the final grade obtained in the course: Round (CG , 1).

Grade of "No Presentado": The final grade in the first or second announcement will be of "No Presentado" in case ☐ and only in case ☐ of not to having written the corresponding final examination.

DATES OF EVALUATION

The calendar of exams approved officially by the Xunta of Centre of the ESEI is published in the following web page:
<https://esei.uvigo.es/docencia/exames/>.

OTHER OBSERVATIONS

REGISTERING FOR THE TESTS OF CONTINUOUS EVALUATION AND EXAMS: For any student, in order to be admitted to taking the midterm exam or any of the final examinations, he or she must register for it through the corresponding online tool within the period established to that end, which will be announced at least 5 days in advance.

ETHICAL CODE AND ETHICAL COMMITMENT:

It is expected of all students an ethical behaviour in all the evaluation tests and exams, in which the answers given by the students should truly reflect the real knowledge and preparation attained in the course. The students must remember that the Estatuto del Estudiante Universitario, in the article 13.2.d), establishes as a duty of students:

"To abstain from using or helping others to use fraudulent procedures in the evaluation tests, in homework or in official documents of the university".

BREAKING OF THE ETHICAL CODE:

One of the types of infractions of the ethical code is cheating or plagiarizing in homework or exams. This is usually detected when in two pieces of work or exams there appear significant coincidences* which would have been wholly impossible without one author having had access to the work of the other or both to an external source. In such cases there will be considered as of equal gravity the fault of whoever had obtained material from someone else as that of whoever allowed someone to have access to his or her own work.

The penalty for an infraction of the ethical code as described above will be the expulsion from the system of continuous evaluation, so that all involved will be evaluated following the criteria for non assistants. In the case that the infraction takes place in a final exam, the penalty will be the qualification of zero in that exam for all involved.

(*) By a significant coincidence or evidence of cheating is understood a phrase or expression of peculiar traits, which is inexplicably repeated identically in different pieces of work or exams by different students and whose repetition none of the involved students is capable of explaining to the satisfaction of the teacher.

Sources of information

Basic Bibliography

David C. Lay, **Linear Algebra and Its Applications**, 978-1292351216, 6 Ed, Addison-Wesley, 2022

Complementary Bibliography

Rodríguez Riotorto, Mario, **Maxima Handbook**, Disponible en <http://maxima.sourceforge.net/docs/manual/es/maxima.pdf>,

Recommendations