## UniversidadeVigo



IDENTIFYING DATA

## Mathematics: Linear algebra



## Competencies

## Code

A2 Students will be able to apply their knowledge and skills in their professional practice or vocation and they will show they have the required expertise through the construction and discussion of arguments and the resolution of problems within the relevant area of study.
B8 Knowledge of the essential subjects and technologies that will allow students to learn and develop new methods and technologies, as well as those that will endow them with versatility to adapt to new situations.
C1 Ability to solve mathematical problems that might arise in engineering. Ability to apply knowledge of: linear algebra; integral and differential calculus; numerical methods; numerical algorithms; statistics and optimization.
D4 Analysis, synthesis and evaluation capacity
D6 Ability to abstract: ability to create and use models that reflect real situations
D11 Critical thinking

| Learning outcomes |  |  |
| :---: | :---: | :---: |
| Expected results from this subject | Training and Learning Results |  |
| RA1. To know how to use gaussian elimination to find an echelon form and the reduced echelon form of a matrix. | A2 B8 | C1 D4 <br>  D6 <br>  D11 |
| RA2. To understand and to know how to solve the questions of existence, uniqueness and universal existence for the systems of linear equations. | $2 \quad B 8$ | C1 D4 <br>  D6 <br>  D11 |
| RA3. To understand the matrix product and its relationship with the composition of linear maps as well as to know its algebraic properties and its applications. | $\text { A2 } \quad \text { B8 }$ | C1 D4 <br>  D6 <br>  D11 |
| RA4. To understand what means for a matrix to have a right inverse, a left inverse or being invertible. | A2 B8 | C1 D4 <br>  D6 <br>  D11 |
| RA5. To understand the concept of determinant of a square matrix, its properties and how to use those properties to calculate a determinant. To know how to calculate a determinant by the method of cofactors. | A2 B8 | C1 D4 <br>  D6 <br>  D11 |
| RA6. To understand the concept of vector space and that of linear map as well as the relationship between the concepts of kernel and image of a linear map and those of null space and column space of a matrix. | A2 B8 | C1 D4 <br>  D6 <br>  D11 |

$\begin{array}{lllll}\text { RA7. To understand the relationship between the questions of universal existence and uniqueness } & \text { A2 } & \text { B8 } & \text { C1 } & \text { D4 }\end{array}$ of solutions of a system of linear equations and the questions of subspace generated by and linear independence of the columns of a matrix, as well as the relationship between those and the properties of surjectivity and inyectivity of a linear map.

| RA8. To find a basis of the null space / column space of a matrix or of the kernel / image space of a A2 | B8 | C1 | D4 |
| :--- | :--- | :--- | :--- |
| linear map. |  | D6 |  |

$\begin{array}{llll}\text { RA9. To find the cartesian equations of a subspace determined by means of generators and to find A2 } & \text { B8 } & \text { C1 } & \text { D4 }\end{array}$
a basis and the cartesian equations of the sum or intersection of two subspaces of $R^{\wedge} n$. D6
$\begin{array}{lllll}\text { RA10. To find the coordinates of a vector with respect to a given basis and to find the change of } & \text { A2 } & \text { B8 } & \text { C1 } & \text { D4 }\end{array}$ coordinates matrix from a given basis to another one.

D6
D11
$\begin{array}{llll}\text { RA11. To know how to use coordinates to translate problems in abstract vector spaces to problems A2 } & \text { B8 } & \text { C1 } & \text { D4 }\end{array}$ in $R^{\wedge} n$.

D6
D11

| RA12. To find the matrix of an endomorphism of a vector space relative to a given basis and to | A2 | B8 | C1 | D4 |
| :--- | :--- | :--- | :--- | :--- |
| know how to determine the effect of a change of basis on the matrix of the endomorphism. |  |  | D6 |  |
|  |  | D11 |  |  |

$\begin{array}{lllll}\text { RA13. To understand the concept of diagonalization of a square matrix and its application to the } & \text { A2 } & \text { B8 } & \text { C1 } & \text { D4 }\end{array}$
calculation of powers of a square matrix and, in general, to the evaluation of a polynomial function D6
on a square matrix.

| RA14. To understand the concept of eigenvector and eigenvalue of a square matrix. | A2 | B8 | C1 |
| :--- | :--- | :--- | :--- |
|  |  | D4 |  |
|  |  | D6 |  |
|  |  | D11 |  |


| RA15. To know how to find the characteristic polynomial of a square matrix, its relationship with | A2 | B8 | C1 | D4 |
| :--- | :--- | :--- | :--- | :--- |
| the eigenvalues and the spectrum of the matrix and the concept of algebraic multiplicity of the |  |  |  |  |
| eigenvalues. |  |  | D11 |  |


| RA16. To know how to find a basis of the eigenspace of an eigenvalue of a square matrix and to | A2 | B8 | C1 | D4 |
| :--- | :--- | :--- | :--- | :--- |
| know how to find a diagonalization of a matrix whose eigenvalues are known. |  |  | D6 |  |


| RA17. To understand the concepts of scalar product and orthogonality in $\mathrm{R}^{\wedge} \mathrm{n}$ and to understand | A2 | B8 | C1 | D4 |
| :--- | :--- | :--- | :--- | :--- |

the null space of a matrix as the orthogonal space to the row space of the matrix. D6

RA18. To calculate the orthogonal projection of a vector on the ray of a nonzero vector and to $\quad$ A2 $\quad$ B8 $\quad$ C1 $\quad$ D4 know how to use such projections to orthogonalize a basis of a subspace of $R^{\wedge} n$ by the Gram- D6

| Schmidt algorithm. |  | D11 |  |
| :--- | :--- | :--- | :--- |
| RA19. To understand the problem of least squares associated with an inconsistent system of linear A2 | B8 | C1 | D4 |

equations and to solve it by means of the corresponding normal equations. D6
$\begin{array}{lllll}\text { RA20. To know the orthogonality properties of the eigenspaces of a symmetric matrix and to know A2 } & \text { B8 } & \text { C1 } & \text { D4 }\end{array}$
how to use them to find an orthogonal diagonalization of a symmetric matrix. D6
$\begin{array}{lllll}\text { RA21. To understand the concept of quadratic form and to know how to represent it by means of a A2 } & \text { B8 } & \text { C1 } & \text { D4 }\end{array}$ symmetric matrix.

D6
$\begin{array}{llllll} & & & \text { D11 }\end{array}$
find its effect on the corresponding symmetric matrix.
D6
RA23. To know how to find a diagonalization of a quadratic form and to know how to use it to $\quad$ A2 $\quad$ B8 $\quad$ C1 $\quad$ D4 classify it and to determine its maximum an minimum values on unit vectors.

## Contents

Topic

| BLOQUE I | SYSTEMS OF LINEAR EQUATIONS: <br> Elementary row operations. <br> Echelon form and Reduced Echelon Form. <br> Vector equations. <br> Matrix equations and homogeneous systems. <br> LINEAR MAPS <br> Linear independence and linear maps. <br> The questions of existence and uniqueness in terms of linear maps. <br> MATRICES: <br> Matrix product. LU factorisation. <br> Invertible matrices. <br> Partitioned matrices. <br> Subspaces and basis. <br> Dimension and Rank. |
| :---: | :---: |
| BLOQUE II | DETERMINANTS: <br> Definition of determinants and cofactors. <br> Calculation by elementary operations. <br> Applications of determinants. <br> VECTOR SPACES: <br> Definition and examples of vector space. <br> Coordinates. Vector subspaces. <br> Linear maps and their associated subspaces. <br> The matrix of a linear map and change os basis. <br> Similar matrices. <br> DIAGONALIZATION: <br> Basic concepts of diagonalization. Eigenvector, eigenvalue and eigenspaces. <br> Characteristic polynomial. <br> Diagonalizable matrices and applications. |
| BLOQUE III | ORTHOGONALITY AND LEAST SQUARES: <br> Inner product spaces and orthogonality. <br> Orthogonal projection on a subspace. The algorithm of Gram-Schmidt. Least Squares problems. <br> SYMMETRIC MATRICES AND QUADRATIC FORMS: <br> Orthogonal diagonalization of symmetric matrices. Quadratic forms. Change of variable and classification. |
| PRÁCTICAS DE LABORATORIO | - Systems of linear equations. <br> - Matrix calculations. <br> - Geometric applications in the plane and in space. <br> - Matrix diagonalization. <br> - Inner product spaces. <br> - Classification of quadratic forms. |
| Planning |  |
|  | Class hoursHours outside the Total hours <br> classroom |
| Lecturing | 24 36 60 |
| Problem solving | 16.5 24.75 41.25 |
| Problem and/or exercise solving | 13 11.75 24.75 |
| Essay questions exam | 321 |
| *The information in the planning table is for guidance only and does not take into account the heterogeneity of the students |  |
| Methodologies |  |
| Descrip |  |
| Lecturing $\begin{array}{l}\text { Lecture } \\ \text { applica }\end{array}$ | ontents by the teacher, illustrated with numerous examples and | given topics. The said problems or exercises will illustrate or will complete the explanation of each lesson.

Simultaneously, there will be proposed exercises and problems that the students will have to solve. The answers will be explained and the qualification obtained by each student will be part of the continuous evaluation.


## Other comments on the Evaluation

The regular assistance to the theoretical and recitatio classes and the degree of participation in them is considered an important part of the course and will be taken into account in the continuous evaluation since it will be key for the realisation of the short answer tests. By regular assistance is meant not reaching 3 unjustified faults of assistance.

The evaluation of any student that assist regularly to class will follow the criteria of continuous evaluation. The rest will be evaluated by the criteria of evaluation for non assisting students.

## CRITERIA OF EVALUATION FOR ASSISTING STUDENTS, 1st EDITION OF RECORDS

The final grade, N , will be calculated by the formula:
$N=0,72 * E+0,28 * P$
where $E$ is the average the the marks, between 0 and 10 , obtained in each the three midterm exams, and $P$ is the average of the marks, between 0 and 10, obtained in the tests given in the recitation classes.

Evaluated competencies: CB1, CG8, CE1, CT4, CT6, CT11.
Evaluated learning outcomes: RA1 to RA23.

## CRITERIA OF EVALUATION FOR NON ASSISTING STUDENTS

Methodology/Single Test: Evaluation of theory and problems.
Description: Written exam that will include evaluation of theoretical concepts and resolution of exercises.
\% Qualification: 100\%.
Evaluated Competences: CB1, CG8, CE1, CT4, CT6, CT11.
Evaluated Learning outcomes: RA1 to the RA23.

## CRITERIA OF EVALUATION FOR 2nd EDITION AND END OF CAREER

The same system of evaluation applied for non assisting students will be used.

## PROCESS OF ASSIGNING THE FINAL GRADE

Independently of the announcement, the final grade will be the symmetrical round to 1 decimal places of the final grade obtained in the course: Round ( $\mathrm{N}, 1$ ).

Grade of "No Presentado": The final grade in the first or second announcement will be of "No Presentado" in case 马and
only in case[ of not to having written the corresponding final examination.

## DATES OF EVALUATION

The calendar of exams approved officially by the Xunta of Centre of the ESEI is published in the following web page: https://esei.uvigo.es/docencia/exames/.

## OTHER OBSERVATIONS

TESTS OF CONTINUOUS EVALUATION AND EXAMS: For any student, in order to be admitted to taking any of the three midterm exams or any of the final examinations, he or she must register for it through the corresponding online tool (in Faitic) within the period established to that end, which will be announced at least 5 days in advance.

## ETHICAL CODE AND ETHICAL COMMITMENT:

It is expected of all students an ethical behaviour in all the evaluation tests and exams, in which the answers given by the students should truly reflect the real knowledge an preparation attained in the course. The students must remember that the Estatuto del Estudiante Universitario, in the article 13.2.d), establishes for the students the duty of:
"To abstain to use or to help in fraudulent procedures in the evaluation tests, in homework or in official documents of the university".

## BREAKING OF THE ETHICAL CODE:

One of the types of infractions of the ethical code is cheating or plagiarizing in homework or exams. This is usually detected when in two pieces of work or exams there appear significant coincidences* which would have been wholly impossible without one author having had access to the work of the other or both to an external source. In such cases there will be considered as of equal gravity the fault of whoever had obtained material from someone else as that of whoever allowed someone to have access to his or her own work.

The penalty for an infraction of the ethical code as described above will be the expulsion form the system of continuous evaluation, so that all involved will be evaluated following the criteria for non assistents. In the case that the infraction takes place in a final exam, the penalty will be thre calification of zero in that exam for all involved.
(*) By a significant coincidence o evidence of cheating is understood a frase or expression of peculiar traits, which is inexplicably repeated identically in different pieces of work or exams and whose repetition none of the involved is capable of explaining to the satisfaction of the teacher.

## Sources of information

## Basic Bibliography

David C. Lay, Linear Algebra and Its Applications, 978-0-321-38517-8, 4 Ed, Addison-Wesley, 2006
Complementary Bibliography
Rodríguez Riotorto, Mario, Maxima Handbook, Disponible en http://maxima.sourceforge.net/docs/manual/es/maxima.pdf,

## Recommendations

## Contingency plan

## Description

=== EXCEPTIONAL MEASURES SCHEDULED ===

## STAGE 1: MIXED TEACHING

In an exceptional situation, in front of the impossibility to give the teaching of a face-to-face way, will use telematic means for the teaching of the classes such like virtual classrooms and explanatory videos.

There will be changes in the system of evaluation, except that the proofs will make *telemáticamente.
STAGE 2: TEACHING NO FACE-TO-FACE
In an exceptional situation, in front of the impossibility to give the teaching of a face-to-face way, will use telematic means for the teaching of the classes such like virtual classrooms and explanatory videos.

There will be changes in the system of evaluation, except that the proofs will make *telemáticamente.
=== ADAPTATION OF THE METHODOLOGIES ===

* EDUCATIONAL METHODOLOGIES THAT KEEP

Methodology 1: Resolution of problems.

* EDUCATIONAL METHODOLOGIES THAT MODIFY

Methodology 1: Lesson *magistral
will make by means of explanatory videos.

* MECHANISM NO FACE-TO-FACE OF ATTENTION To THE STUDENTS (*TUTORÍAS)
will make by means of the use of the email and the tool Remote Campus.
* THERE ARE not OTHER MODIFICATIONS
$===$ ADAPTATION OF THE CONTINUOUS EVALUATION $===$
* PROOFS THAT KEEP

Proof 1: [previous Weight 24\%] [Weight Proposed 14\%]
Description: Examination of an hour of length in which it will be necessary to resolve two or three problems.
Competitions evaluated: Subject 1 (Systems of linear equations), Subject 2 (linear Applications) and Subject 3 (Matrices).
It tests 2: [previous Weight 24\%] [Weight Proposed 14\%]
Description: Examination of an hour of length in which it will be necessary to resolve two or three problems.
Competitions evaluated: Subject 4 (Determinants), Subject 5 (Vectorial Spaces) and Subject 6 (*Diagonalización).
It tests 3: [previous Weight 24\%] [Weight Proposed 14\%]
Description: Examination of an hour of length in which it will be necessary to resolve two or three problems.
Competitions evaluated: Subject 7 (*Ortogonalidad) and Subject 8 (quadratic Forms).

10 you Test type test (weekly) [previous Weight 28\%] [Weight Proposed 28\%]

* PROOFS THAT DELETE

Any

* NEW PROOFS

Work 1: [Weight 10\%]
Description: Work to make home and to deliver by means of consistent Remote Campus in the resolution of a problem advanced or in the development of a subject of the program.
Competitions evaluated: Subject 1 (Systems of linear equations), Subject 2 (linear Applications) and Subject 3 (Matrices).
Work 2: [Weight 10\%]
Description: Work to make home and to deliver by means of consistent Remote Campus in the resolution of a problem advanced or in the development of a subject of the program.
Competitions evaluated: Subject 4 (Determinants), Subject 5 (Vectorial Spaces) and Subject 6 (*Diagonalización).

Work 3: [Weight 10\%]
Description: Work to make home and to deliver by means of consistent Remote Campus in the resolution of a problem advanced or in the development of a subject of the program.
Competitions evaluated: Subject 7 (*Ortogonalidad) and Subject 8 (quadratic Forms).
$===$ ADAPTATION OF THE EVALUATION IN MODALITY NO FACE-TO-FACE $===$
Unchanged.

